

14-1 HINGE RESTRAINER DESIGN METHOD

Introduction

Bridge Design Aid 14-1 provides a simplified method for designing cable restrainers for bridges. It is based on a research study using a simplified two-degree-of-freedom linear analytical model. Twenty six different earthquake records were utilized and verified through non-linear analysis using ground motion records scaled to $0.7g$ ¹. Some of the advantages of this method are:

- Uses a simplified two-degree-of-freedom linear analytical model
- Single-step process - no iterations required
- No modal analysis required
- Conservative for most known earthquake ground motions
- Simplified ground motion period (T_g) determination compatible with the Seismic Design Criteria (SDC) soil types

Assumptions:

- The effects of friction are negligible
- The skew is ≤ 30 degrees. For bridges with skews > 30 degrees, the lead office should consult with the Office of Earthquake Engineering (OEE) for guidance.
- The hinge is represented by a linearized model as shown in Figure 1:

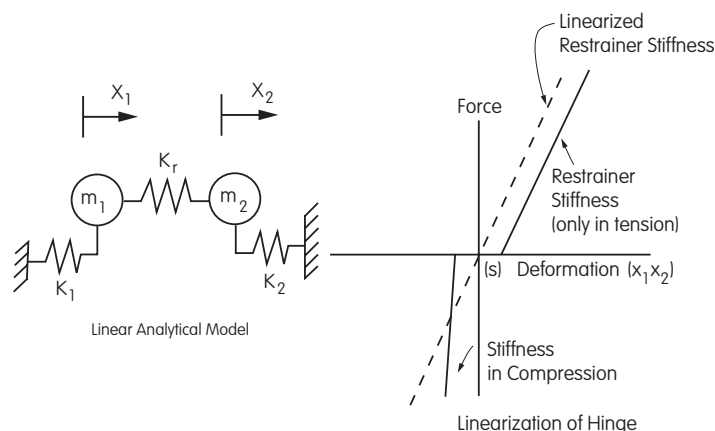


Figure 1

¹. UC Berkeley Report No. UCB/EERC 97/12. "New Design and New Analysis Procedures for Intermediate Hinges in Multiple - Frame Bridges."

Definitions of Variables

- A = Restrainer cross-sectional area
 c = The effective modal damping factor
 c_{avg} = The average effective modal damping factor of frames 1 and 2
 c_i = The effective modal damping factor for frame i
 D_1 = The displacement demand of the less flexible frame
 D_2 = The displacement demand of the more flexible frame
 D_{eq0} = The unrestrained relative hinge displacement
 D_i = The spectral displacement demand of frame i
 D_{ia} = The spectral displacement of frame i adjusted by a damping reduction factor R_{di}
 D_r = The restrained relative hinge displacement
 D_y = The yield (max) elongation the restrainer is expected to experience in a seismic event
 E = Restrainer modulus of elasticity
 f = An effective stiffness adjustment factor
 F = Adjusted effective stiffness factor
 F_{eff} = Effective stiffness factor
 F_{yi} = Yield force of frame i
 G = Soil shear modulus
 G_{max} = Maximum soil shear modulus
 H = Soil depth
 K_1 = The stiffness of the less flexible frame
 K_2 = The stiffness of the more flexible frame
 K_{1eff} = The effective stiffness of the less flexible frame
 K_{2eff} = The effective stiffness of the more flexible frame
 K_{ieff} = The effective stiffness of the frame i
 K_i = The stiffness of the frame i
 K_{mod} = The reciprocal of the sum of flexibilities of frames 1 and 2
 K_r = The restrainer stiffness

- K_{min} = The minimum restrainer stiffness required if unseating is possible during the elastic stage
 l = Length of restrainer
 L = Relative hinge displacement limit
 m_i = Mass of frame “i”
 N = Number of restrainers
 r = An adjustment factor for R
 R = Restraint level factor
 R_d = Displacement reduction factor (SDC 2.1.5)
 R_{di} = Displacement reduction factor for frame i (SDC 2.1.5)
 s = Restrainer slack
 T_1 = The fundamental (natural) period of the less flexible frame
 T_2 = The fundamental (natural) period of the more flexible frame
 T_g = The ground period
 T_{1eff} = The effective fundamental (natural) period of the less flexible frame
 T_{2eff} = The effective fundamental (natural) period of the more flexible frame
 T_{ieff} = The effective fundamental (natural) period of frame i
 T_i = The fundamental (natural) period of frame i
 V_s = The shear wave velocity
 β = The effective period ratio T_{2eff} / T_{1eff}
 Δy_i = Yield displacement of frame i
 μ_d = The expected displacement ductility demand, default = 4.0
 μ_{avg} = Average displacement ductility demand of frames 1 and 2
 ρ_{12} = Modal cross-correlation coefficient

Design Theory:

The restrainer stiffness, K_r required for controlling the unrestrained relative hinge displacement D_{eq0} , may be determined by:

$$K_r = R F K_{mod.} \quad \text{Eq. (1.1)}$$

The value of the restrained relative hinge displacement, D_r , should be selected based on the purpose of restrainers such as:

- Protect the bearings and seals of the bridge in a moderate event,
- Prevent unseating from a short seat where seat extensions are not practical,
- Control the response of the structure in a major event.

This method utilizes the effective stiffness concept to determine the effective period and spectral displacements of the yielding frames:

For K_{eff} (effective stiffness), see Figure 2

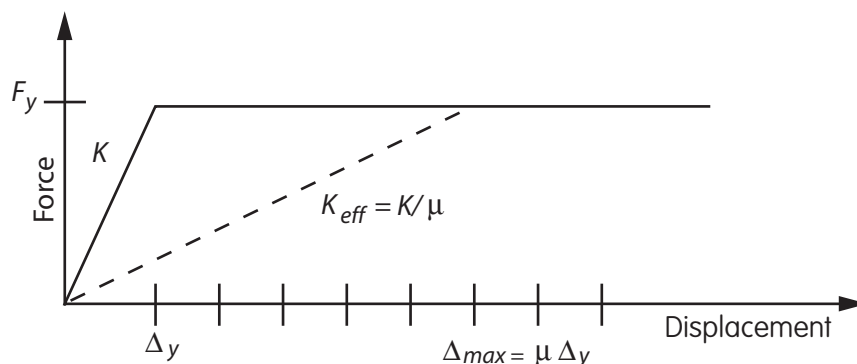


Figure 2

Hinge Restrainer Design Procedure

Determine N

1) Determine D_{eq0} :

For all frames “ i ” determine

m_i (slugs/in)

K_i (kips/in)

$$T_i = 2\pi\sqrt{(m_i / K_i)} \quad (\text{sec}) \quad \text{Eq. (1.2)}$$

Δy_i (in)

μ_d and F_{yi} (kips)

For all frames “ i ” calculate effective values:

$$K_{ieff} = K_i / \mu_d \quad \text{Eq. (1.3)}$$

$$T_{ieff} = 2\pi\sqrt{(m_i / K_{ieff})} \quad \text{Eq. (1.4)}$$

Note: if μ_d is not known use a default value of 4.0

For all frames “ i ” determine D_i (from ARS curves ²), c_i , R_{di} , and D_{ia} :

D_i = Spectral Displacement based on (T_{ieff})

$$c = 0.05 + [1 - (0.95/\sqrt{\mu}) - 0.05\sqrt{\mu}] / \pi \quad (\text{See Figure 3}) \quad \text{Eq. (1.5)}$$

$$R_{di} = [1.5 / (40 c + 1)] + 0.5 \quad (\text{See Figure 3}) \quad \text{Eq. (1.6)}$$

$$D_{ia} = R_{di} D_i \quad \text{Eq. (1.7)}$$

For adjacent frames 1 and 2 determine:

$$\beta = T_{2eff} / T_{1eff} \quad \text{Eq. (1.8)}$$

$$\rho_{12} = \frac{8c_{avg}^2 \sqrt{(1+\beta)^3}}{(1-\beta^2)^2 + 4c_{avg}^2 \beta(1+\beta)^2} \quad (\text{See Figure 4}) \quad \text{Eq. (1.9)}$$

² ARS curves are for 5% damping. Spectral displacements should be adjusted for other values.

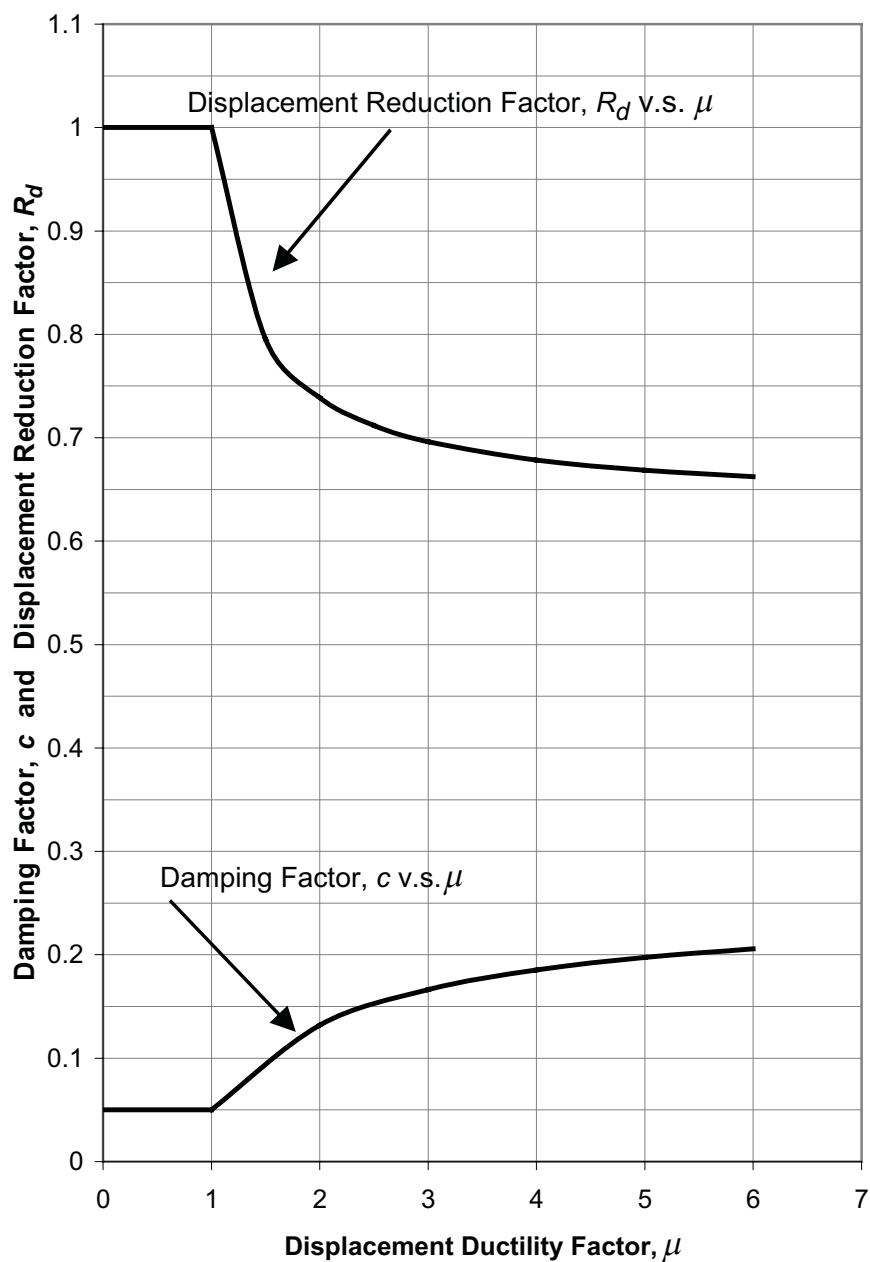


Figure 3

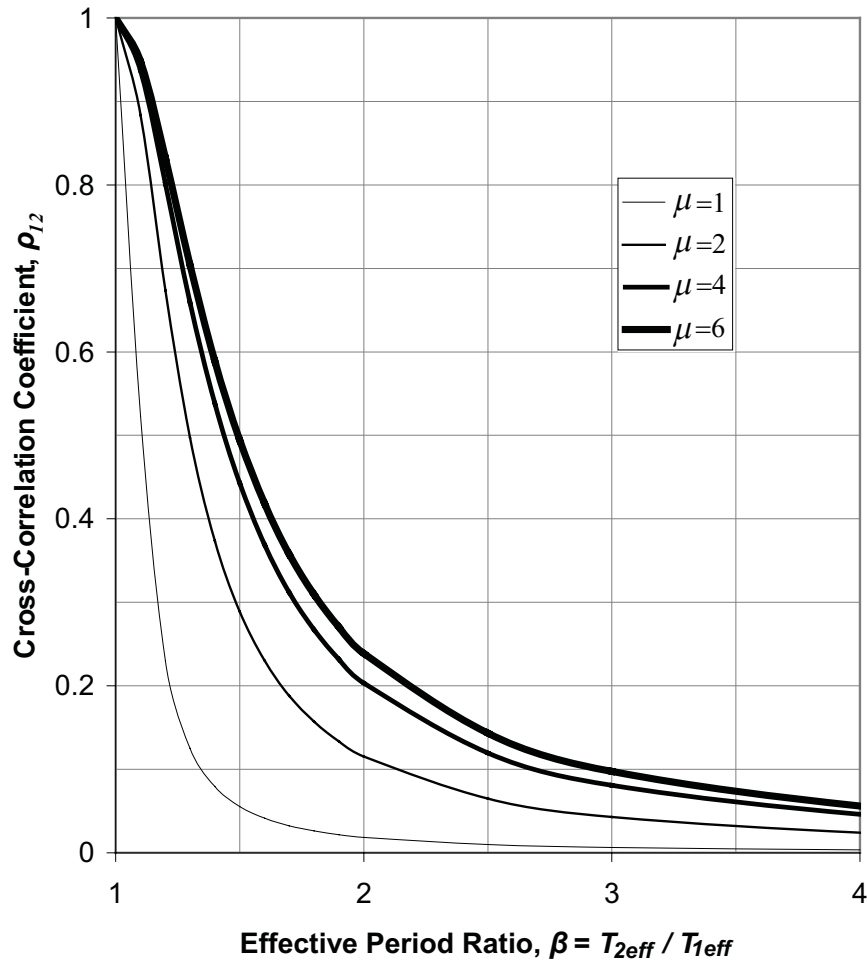


Figure 4

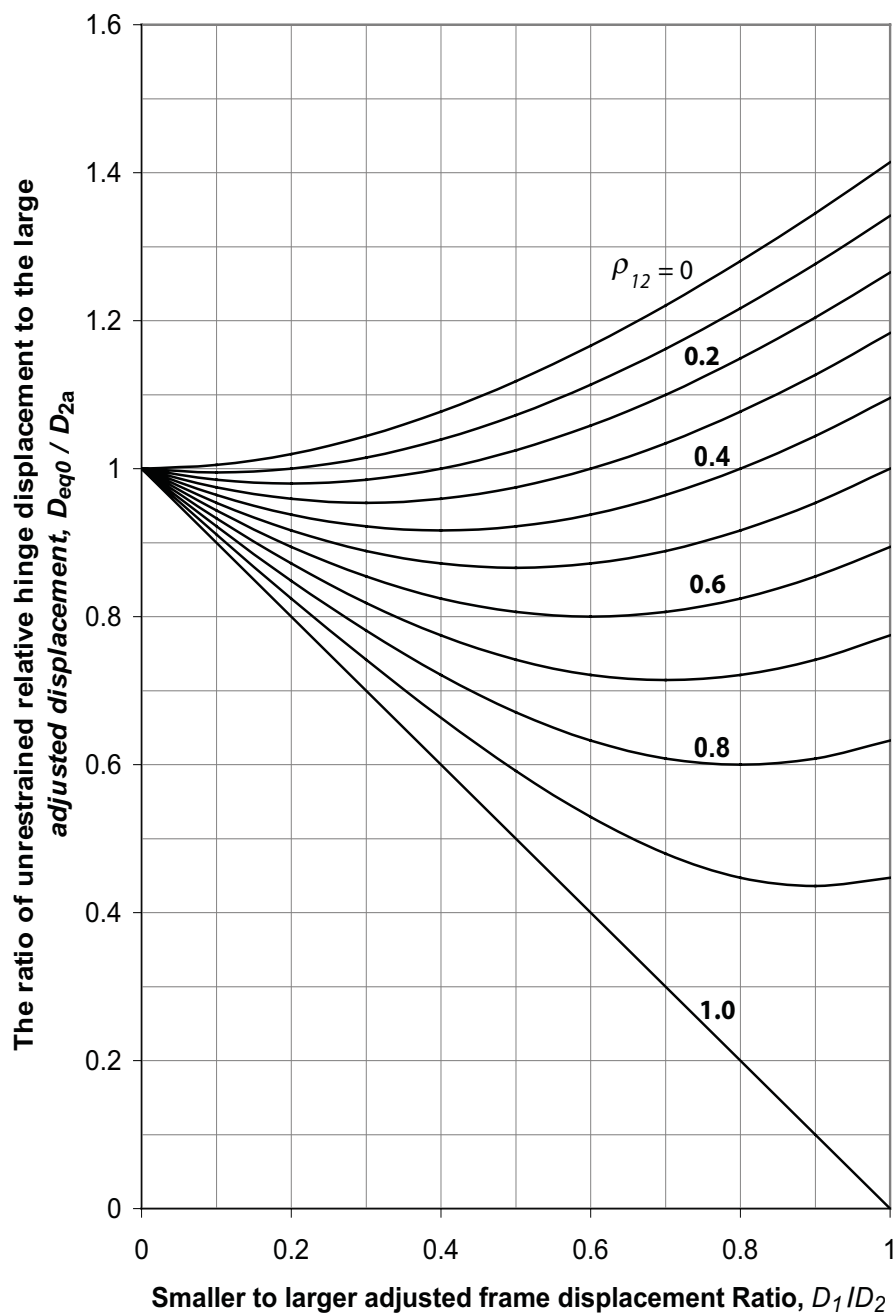


Figure 5

Calculate D_{eq0} :

$$D_{eq0} = \sqrt{(D_{1a}^2 + D_{2a}^2 - 2\rho_{12}D_{1a}D_{2a})} \quad (\text{See Figure 5}) \quad \text{Eq. (1.10)}$$

2) Determine R

Find the allowable or desired restrained relative hinge displacement, D_r

$$D_r = D_y + s \quad \text{Eq. (1.11)}$$

Calculate the displacement limit, L

$$L = D_r / D_{eq0} \quad \text{Eq. (1.12)}$$

Calculate r ,

$$r = -L + 1.5 \quad (\text{See Figure 6}) \quad \text{Eq. (1.13)}$$

Calculate R ,

$$R = r (1 - 1.66L + 0.67/L) \quad (\text{See Figure 6}) \quad \text{Eq. (1.14)}$$

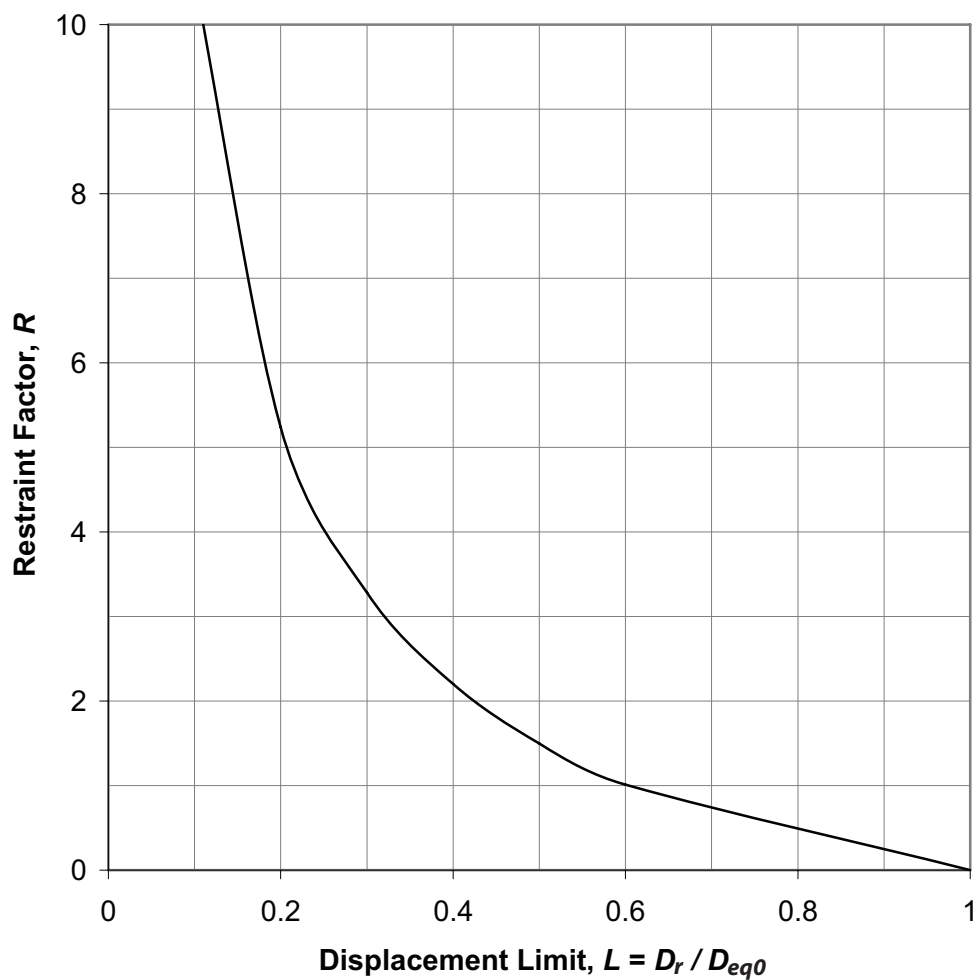


Figure 6

3) Determine F

Determine T_g ,

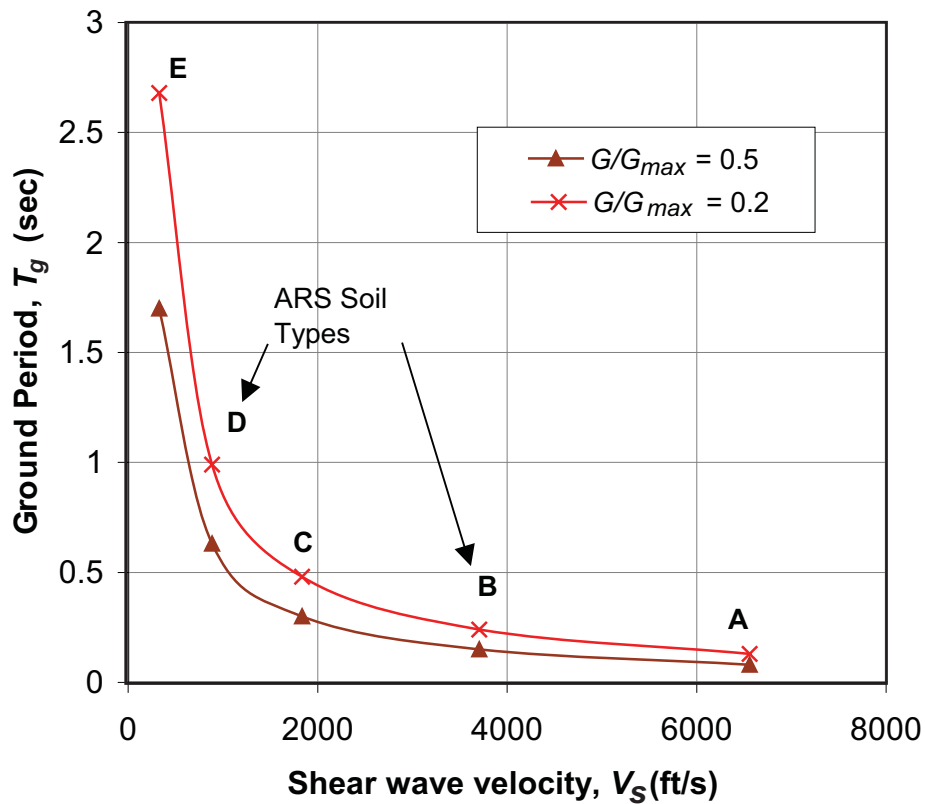
$$T_g = 4H / \left[V_s \sqrt{(G / G_{\max})} \right] \quad (\text{See Figure 7}) \quad \text{Eq. (1.15)}$$

Calculate T_2 / T_g

Determine F_{eff} (See Figure 8)

Determine the adjustment factor, f (See Figure 9)

Calculate $F = f \times F_{\text{eff}}$



Note: G/G_{\max} varies inversely to ground motion intensity

Figure 7

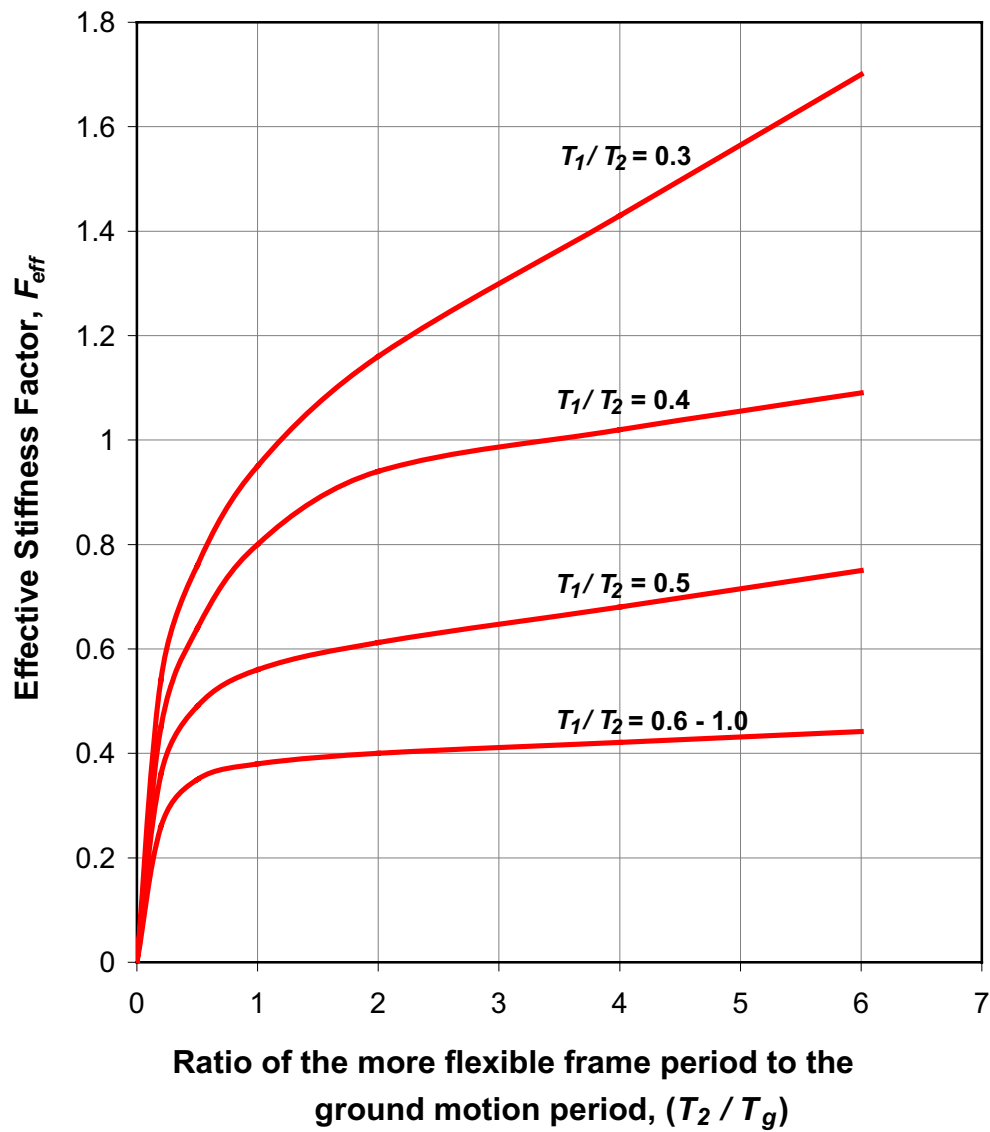


Figure 8

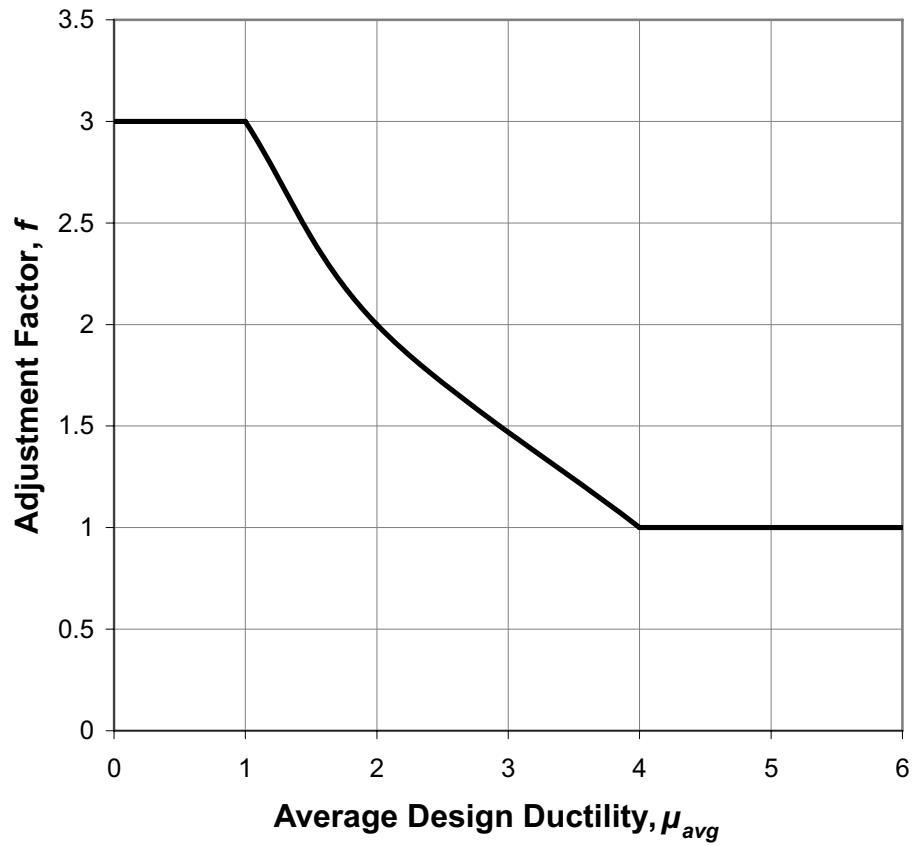


Figure 9

4) Determine K_{mod}

Calculate $K_{mod} = K_1 K_2 / (K_1 + K_2)$ (See Figure 10)

Eq. (1.16)

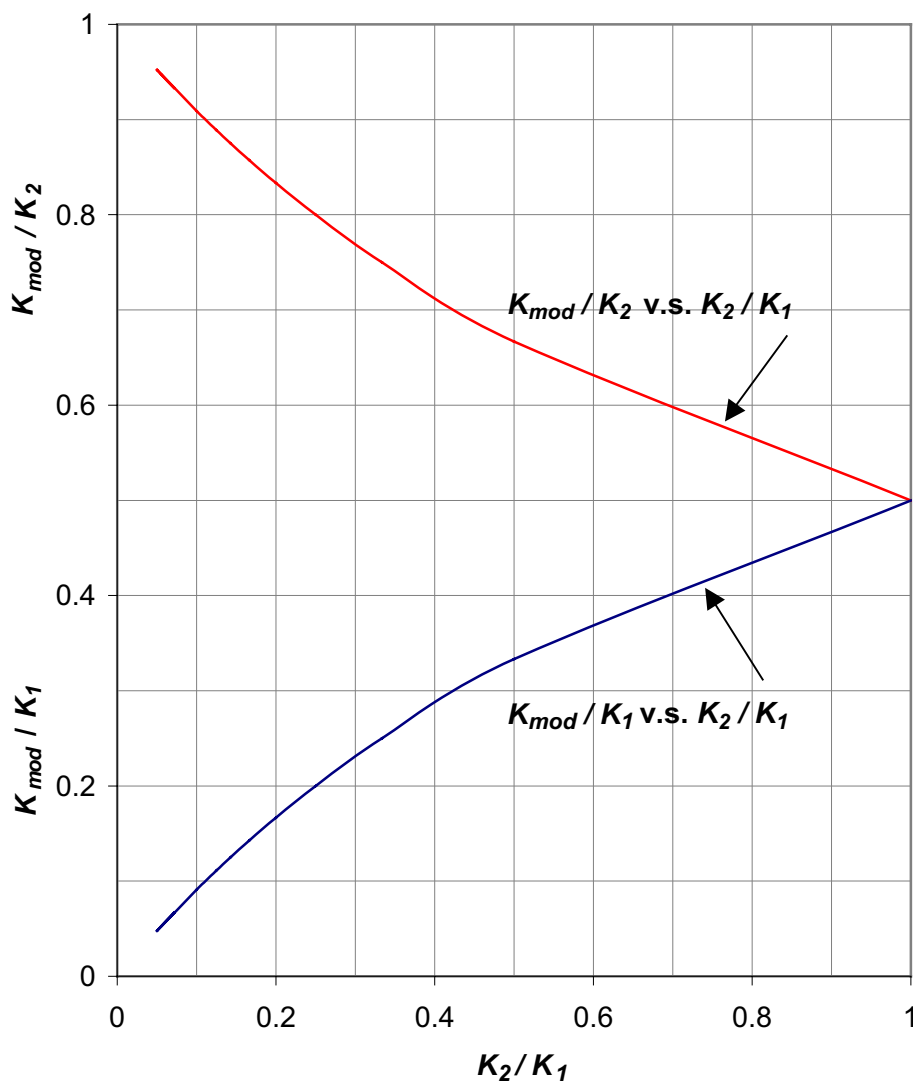


Figure 10

- 5) Calculate the required restrainer stiffness K_r using Eq. (1.1)
- 6) Check if K_{rmin} requirement applies

If $D_{eq0} - D_y > \Delta_{y2}$ then continue, otherwise, proceed to Step 7 Eq. (1.17)

Calculate $K_{rmin} = F_{y2} / D_y$

If $K_{rmin} > K_r$ calculated in step 5 above then K_{rmin} controls and

use $K_r = K_{min}$

If $K_r > K_{min}$ then use K_r

7) Calculate the number of restrainers, N

$$N = K_r l / EA \quad \text{Eq. (1.18)}$$

Restrainer Design for Multi-Span Simply Supported Bridges

The simple supports of multi-span simply supported bridges (SSB) can be modeled similar to in-span hinges of multi-frame structures. The required restrainer stiffness may be determined using the same method.

To model a simple support of a SSB, each joint should be considered individually (Note that every internal support contains two joints, one for each simple span it supports). Assume that the selected joint is the only joint that allows movement (hinge) while all other joints act as pins. This results in two frames, one to the left, and the other to the right of the selected joint (hinge). This will include several scenarios, with frames defined on either side of the hinge. The number of spans and columns included in each frame is determined by the joint modeling procedure shown in the next section.

For typical bridges assume it is not necessary to consider more than two spans and two columns on either side of the hinge for any particular scenario (see Figure 11). This results in a maximum of four scenarios for every hinge. For each scenario, the stiffnesses, periods, displacement demands, hinge opening and the corresponding required restrainer stiffness are determined in the same manner as for in-span hinges. The scenario that results in the largest hinge opening will control. If the hinge opening demand is greater than or equal to the available seat length, restrainers must be provided as required or may be added for performance³. The same procedure is repeated for the second joint of the same support and the required restrainer stiffness is calculated. Apply the same procedure to all other supports. Abutments should be considered fixed.

³. Memo to Designers 20-4 requires retrofit for all seat lengths ≤ 12 inches.

Joint Modeling Procedure Illustration:

The structure in Figure 11 illustrates the joint modeling procedure:

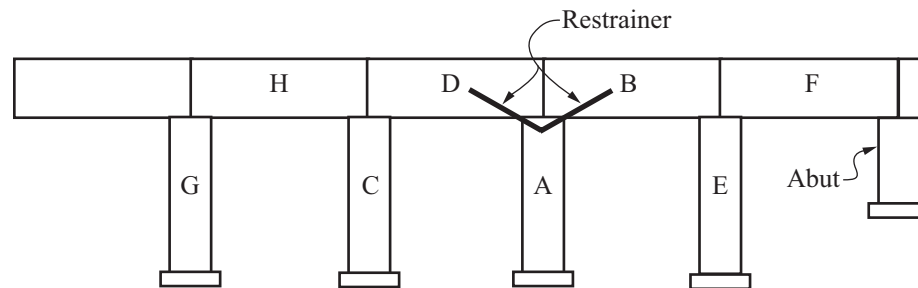


Figure 11

- 1.) For retrofits, if the support is a pin then the strength of the bearing must be evaluated and if the strength is greater than the earthquake force then no restrainers are required to prevent unseating. Restrainers may be added if performance or bearing protection is required. If the support bearing strength is less than the seismic force then it should be treated as a roller.
- 2.) To design restrainer AD consider the two frames:

Scenario 1:

Frame AB (column A and span B) moving right

Frame CD (column C and span D) moving left

Calculate the periods of both frames AB and CD and the required restrainer stiffness utilizing the same method for intermediate hinges of multi-span bridges.

Similarly check other scenarios for tension and compression models (no need to check more than two columns and spans on each side of the joint)

Scenario 2:

Frame ABEF moving right

Frame CD moving left

Scenario 3:

Frame AB moving right

Frame CDGH moving left

Scenario 4:

Frame ABEF moving right

Frame CDGH moving left

The situation that results in the most restrainers controls the design at that particular joint (hinge)

- 3.) To design restrainer AB investigate the two frames:

Frame AD (column A and span D) moving left

Frame BE (column E and span B) moving right

Calculate the periods of both frames AD & BE, the required restrainer stiffness and check other tension and compression models similar to restrainer AD.

- 4.) Repeat steps 2 and 3 for all intermediate supports.

- 5.) For Abutment supports (end spans):

Always model the abutment as fixed and T_1/T_2 may be assumed to be 0.3 for simplicity since it represents a relatively very rigid frame (abutment like) adjacent to a flexible frame and the charts will be applicable without further analysis.

Scenario 1:

Frame 1 = Abutment (fixed)

Frame 2 = Frame EF (moving left)

Scenario 2:

Frame 1 = Abutment (fixed)

Frame 2 = Frame EFAB (moving left)

- 6.) For the restrainer EF, the frame consisting of span F and the abutment shall be considered fixed and treated as the Frame 1 above.

Scenario 1:

Frame 1 = Abutment + span F (fixed)

Frame 2 = Frame EB (moving left)

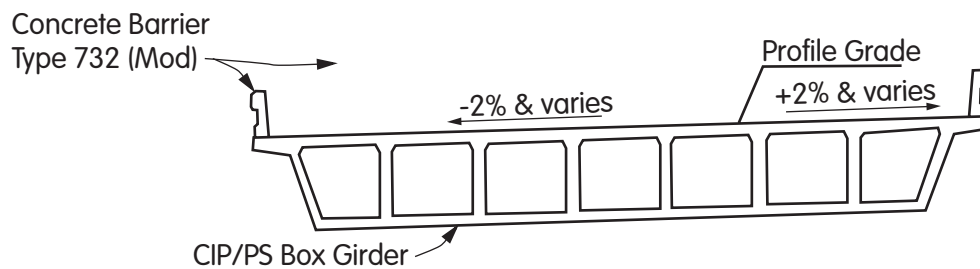
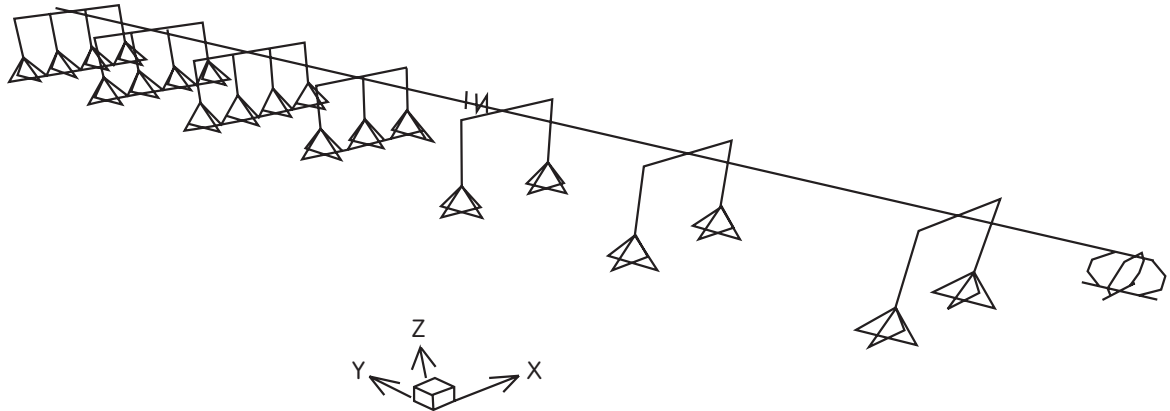
Scenario 2:

Frame 1 = Abutment + span F (fixed)

Frame 2 = Frame EBAD (moving left)

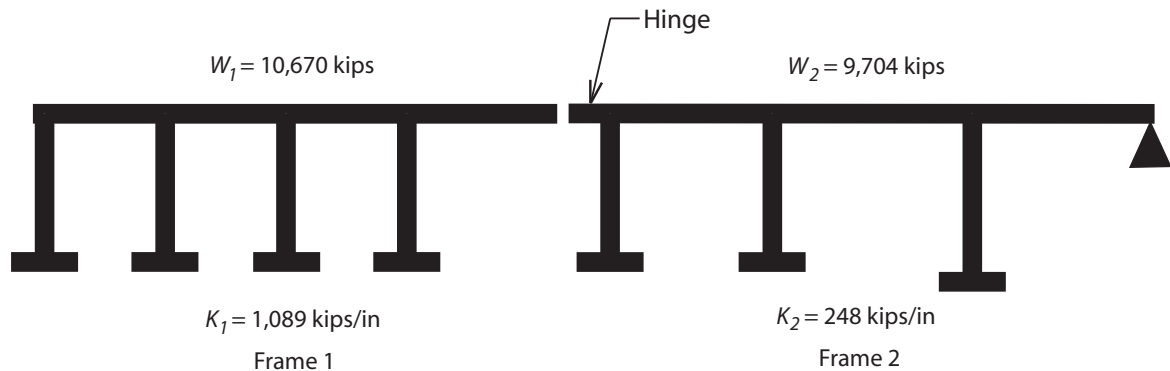
Example Calculation for In-Span Hinges:

Determine the number of restrainers required for the following structure:



Given:

The structure is in a 0.7g seismic zone with a type C soil as described in the Caltrans Seismic Design Criteria (SDC). The structure has 6 inch hinge seats with a 4 inch allowable movement. The following sketch summarizes additional given information for each frame:



- 1) Determine D_{eq0} :

For all frame, determine: m_i , K_i , T_i , D_i (ARS curves), Δy_i , μ_d , and F_{yi}

$$m_1 = 27.61 \text{ slugs/in}$$

$$m_2 = 25.11 \text{ slugs/in}$$

$$K_1 = 1089 \text{ kips/in}$$

$$K_2 = 248 \text{ kips/in}$$

$$T_1 = 2\pi(27.61 / 1089) = 1 \text{ sec}$$

$$T_2 = 2\pi(25.11 / 248) = 2 \text{ sec}$$

$$\Delta y_1 = 3.36 \text{ inches (given)}$$

$$\Delta y_2 = 4.17 \text{ inches (given)}$$

$$\mu_{d1} = 3.59 \text{ (given)}$$

$$\mu_{d2} = 5.34 \text{ (given)}$$

$$F_{y1} = 1089(3.36) = 3,659 \text{ kips}$$

$$F_{y2} = 248(4.17) = 1,034 \text{ kips}$$

Apply Eq. (1.3) calculate K_{ieff}

Apply Eq. (1.4) calculate T_{ieff}

$$K_{1eff} = 1,089 / 3.59 = 303.3 \text{ kips/in}$$

$$T_{1eff} = 2\pi(27.61 / 303.3) = 1.9 \text{ sec}$$

$$K_{2eff} = 248 / 5.34 = 46.4 \text{ kips/in}$$

$$T_{2eff} = 2\pi\sqrt{(25.11 / 46.4)} = 4.6 \text{ sec}$$

For all frame "i" determine D_i (ARS curves), c_i , R_d , and D_{ia} :

D_1 = Spectral Displacement (at 1.9 sec) = 21 in

D_2 = Spectral Displacement (at 4.6 sec) = 32 in

Apply Eq. (1.5) in Figure 3:

$$c_1 = 0.05 + \left[1 - \left(0.95 / \sqrt{3.59} \right) - 0.05 \sqrt{3.59} \right] / \pi = 0.18$$

$$c_2 = 0.05 + \left[1 - \left(0.95 / \sqrt{5.34} \right) - 0.05 \sqrt{5.34} \right] / \pi = 0.20$$

Apply Eq. (1.6) in Figure 3:

$$R_{d1} = [1.5 / (40(0.18) + 1)] + 0.5 = 0.68$$

$$R_{d2} = [1.5 / (40(0.20) + 1)] + 0.5 = 0.67$$

Apply Eq. (1.7):

$$D_{1a} = 0.68 (21) = 14.3 \text{ in}$$

$$D_{2a} = 0.67 (32) = 21.4 \text{ in}$$

For adjacent frames 1 and 2 determine β and ρ_{12}

Apply Eq. (1.8):

$$\beta = 4.6 / 1.9 = 2.42$$

Calculate c_{avg} :

$$c_{avg} = (0.18 + 0.20) / 2 = 0.19$$

Apply Eq. (1.9) in Figure 4:

$$\rho_{12} = \frac{8(0.19)^2 (1+2.42)(2.42)^{3/2}}{(1-2.42^2)^2 + 4(0.19)^2 (2.42)(1+2.42)^2} = 0.134$$

Calculate the unrestrained relative hinge displacement, D_{eq0}

Apply Eq. (1.10), Figure 5:

$$D_{eq0} =$$

2) Determine R :

Calculate D_r , L , r , and R

Apply Eq. (1.11):

$$D_r = 3 + 1 = 4 \text{ in}$$

Apply Eq. (1.12):

$$L = 4 / 24.1 = 0.166$$

Apply Eq. (1.13), Figure 6:

$$r = -0.166 + 1.5 = 1.33$$

Apply Eq. (1.14), Figure 6:

$$R = 1.33 (1 - 1.66(0.166) + 0.67/0.166) = 6.33$$

3) Determine F :

Determine T_g , F_{eff} , f , and F :

From Figure 7 (assume high ground motion intensity $G/G_{max} = 0.2$):

$$T_g = 0.5 \text{ sec}$$

Calculate T_2 / T_g and T_1 / T_2 :

$$T_2 / T_g = 2 / 0.5 = 4$$

$$T_1 / T_2 = 1 / 2 = 0.5$$

Determine F_{eff} :

From Figure 8, $F_{eff} = 0.68$

Determine f :

Average $\mu_d = (3.59 + 5.34) / 2 = 4.47 > 4.0$

From Figure 9, $f = 1.0$

Calculate F :

$$F = f \times F_{eff} = 1.0 \times (0.68) = 0.68$$

- 4) Determine K_{mod}

Apply Eq. (1.16), Figure 10:

$$K_{mod} = 1089 (248) / 1089 + 248 = 202 \text{ kips/in}$$

- 5) Determine the required restrainer stiffness, K_r

Apply Eq. (1.1):

$$K_r = 6.33 (0.68)(202) = 870 \text{ kips/in}$$

- 6) Check if K_{rmin} requirement applies

Apply Eq. (1.17):

$$D_{eq} - D_y = 24.1 - 3 = 21.1 > \Delta y_2 = 4.17 \therefore$$

K_{rmin} applies

$$\text{Calculate } K_{rmin} = F_{y2} / D_y$$

If $K_{rmin} > K_r$ calculated in step 5 above then K_{rmin} controls

$$K_{rmin} = 1034 / 3 = 345 \text{ kips/in} < 870 \text{ ok use } K_r$$



- 7) Determine the required number of restrainers, N

Apply Eq. (1.18)

$$N = 870 \text{ kips/in (240 in)} / 14000 \text{ ksi (0.222 in}^2\text{)} = 67.2 \text{ say 68 cables}$$

References:

1. California Department of Transportation, Bridge Memo to Designers 20-4
2. UC Berkeley Report No. UCB/EERC 97/12 - "New Design and New Analysis Procedures for Intermediate Hinges in Multiple - Frame Bridges."